6D Cooling Simulations for the Muon Collider

Pavel Snopok
University of California Riverside

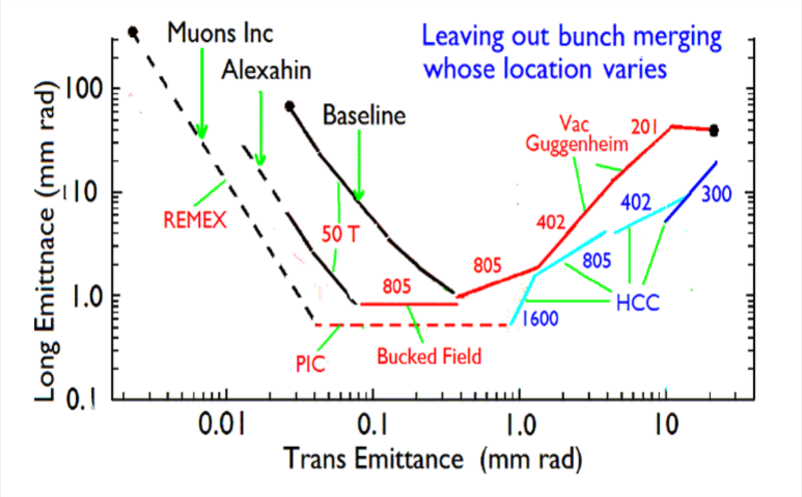
Guggenheim Workshop @ Fermilab

1. Muon Collider vs. Neutrino Factory cooling needs
2. Muon Cooling scheme for the Muon Collider
3. 6D cooling and emittance exchange
4. Cooling lattices
5. RF in magnetic field
6. Open cavity lattice simulations
7. Summary

Muon Collider cooling needs

- Neutrino Factory:
  - Might be feasible with no cooling.
  - Some transverse cooling is cost-effective.
  - Virtually no longitudinal cooling.
  - Overall 6D emittance reduction of approximately one order in magnitude.
- Muon Collider:
  - Strong fully 6D cooling.
  - 6D emittance reduction factor of 10^6.
  - Proposed cooling schemes—next slide.

Muon Collider cooling scheme



6D cooling and emittance exchange

Combining Cooling and Heating:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_\perp}{2}\frac{d\langle\theta_{rms}{}^2\rangle}{ds}$$

- **Low-Z absorbers (H_2, Li, Be, …) to reduce multiple scattering**
- **High Gradient RF**
  - **To cool before μ-decay (2.2γ μs)**
  - **To keep beam bunched**
- **Strong-Focusing at absorbers**
  - **To keep multiple scattering**
  - **less than beam divergence …**

⇒ Quad focusing ?

⇒ Li lens focusing ?

⇒ Solenoid focusing?

$$\frac{d\langle\theta_{rms}^2\rangle}{ds} = \frac{z^2 E_s^2}{\beta^2 c^2 p_\mu^2 L_R}$$

Emittance Exchange

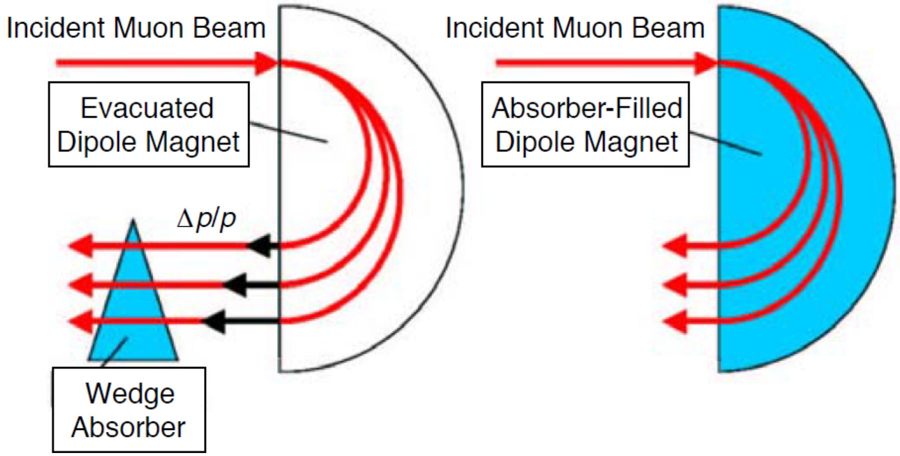


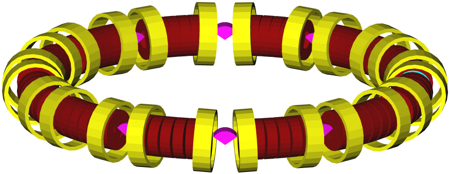
Image courtesy of Muons, Inc.

Cooling lattices

Various cooling proposals

- RFOFO ring / Guggenheim helix
- modification: Open cavity lattice
- Helical cooling channel (Muons, Inc.)
- FOFO Snake (Y. Alexahin)
- Quadrupole and dipole rings

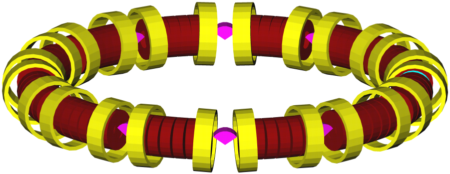
RFOFO ring



RFOFO ring

- Yellow: tilted magnetic coils generating required bending and dispersion.
- Purple: wedge absorbers for cooling and emittance exchange.
- Red/Brown: RF cavities to restore energy lost in the absorber (longitudinal direction only).

RFOFO ring



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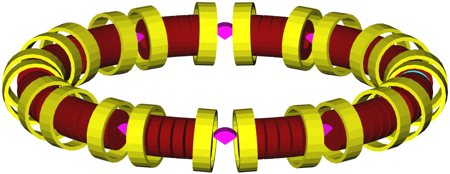

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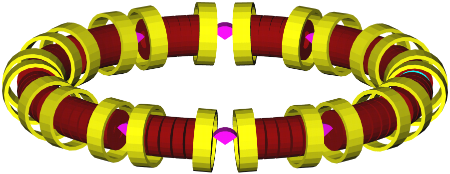

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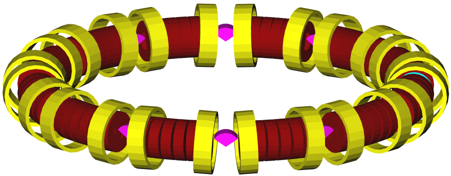
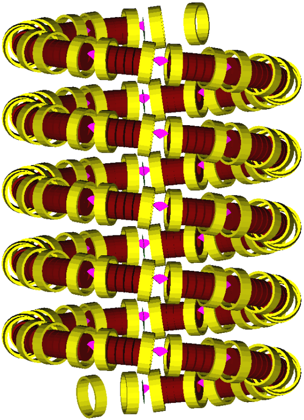
RFOFO ring and Guggenheim helix



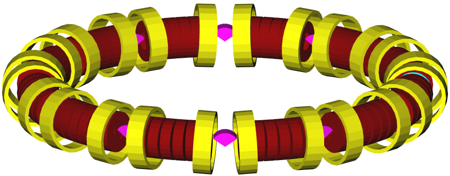
RFOFO ring

- Advantages: fast cooling, compact design, RF reuse.
- Challenges: absorber overheating, injection/extraction, continuous operation.



RFOFO-based Guggenheim helix

RFOFO ring and Guggenheim helix



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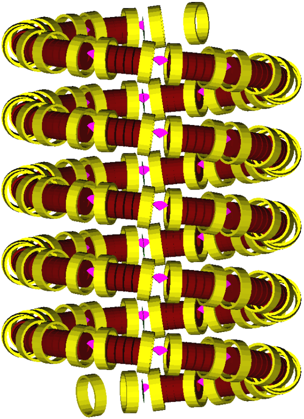

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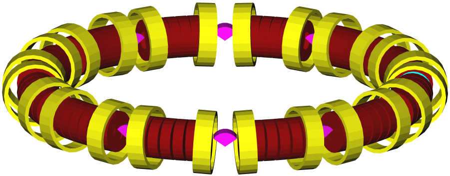

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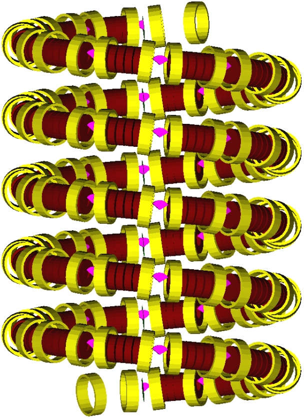
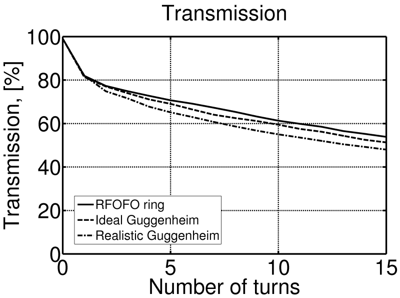


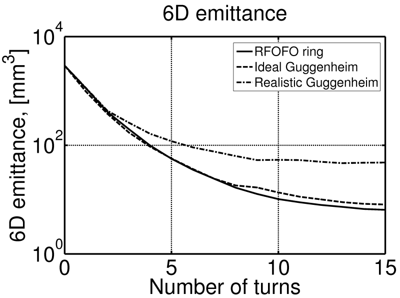
RFOFO-based Guggenheim helix

Parameter comparison

	RFOFO	Guggenheim
Circumference, [m]	33.00	33.00
RF frequency, [MHz]	201.25	201.25
RF gradient, [MV/m]	12.835	12.621
Maximum axial field, [T]	2.77	2.80
Pitch, [m]	0.00	3.00
Pitch angle, [deg]	0.00	5.22
Radius, [mm]	5252.113	5230.365
Coil tilt (wrt orbit), [deg]	3.04	3.04
Average momentum, [MeV/c]	220	220
Reference momentum, [MeV/c]	201	201
Absorber angle, [deg]	110	110
Absorber thickness on beam axis, [cm]	27.13	27.13

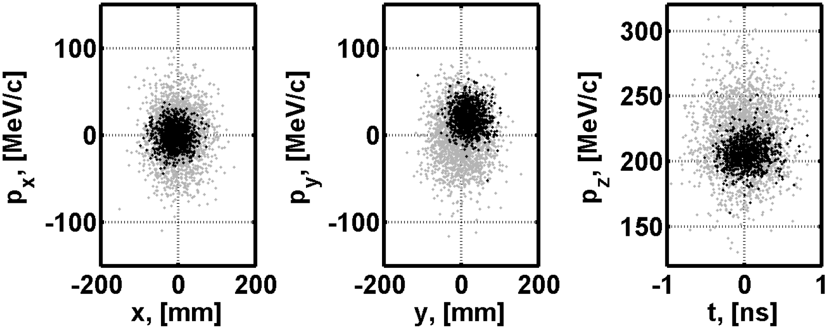
Performance studies





- 6D emittance is reduced by a factor of 448 in the RFOFO ring or a factor of 360 in the Guggenheim helix (495 m) with no windows
- or by a factor of 60 with windows in the RF cavities and absorbers

Phase space reduction



Solenoidal lattices

All the solenoidal cooling lattices

- helical cooling channel,
- FOFO snake,
- RFOFO ring and Guggenheim helix

have one important thing in common:

- they use solenoids to focus and bend particles, generate dispersion;
- RF cavities operate in the strong magnetic field.

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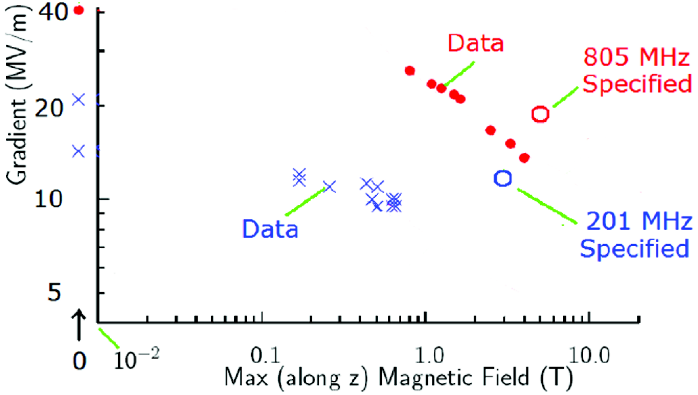
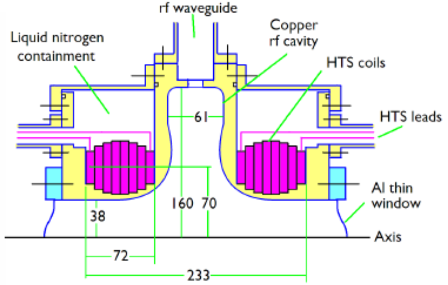
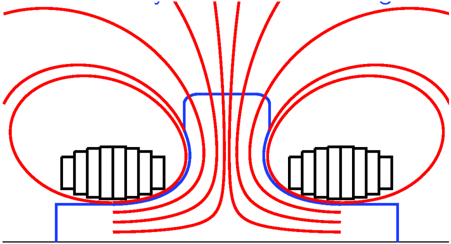
Experimental results

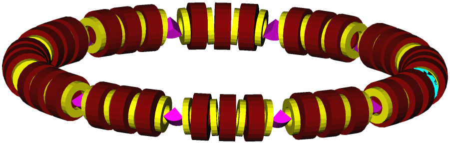


Open cavity lattice

Magnetic insulation







- Open cavity lattice
- Coils in the irises
- Coils are tilted to generate bending field

Open cavity and RFOFO parameters compared

Parameter	Unit	Open cavity	RFOFO
Number of cells		12	12
Circumference	[m]	30.72	33.00
Radius	[m]	4.889	5.252
RF frequency	[MHz]	201.25	201.25
RF gradient	[MV/m]	16.075	12.835
Maximum axial field	[T]	3.23	2.80
Reference momentum	[MeV/c]	214	201
Coil tilt	[deg]	4.90	3.04
Number of coils per cell		4	2
Current densities	[A/mm^2]	[63,45,-45,-63]	[95,-95]
Number of RF cavities		3	6
Length of each RF cavity	[mm]	385	282.5
Absorber angle	[deg]	90	110
Absorber vertical offset	[cm]	12.0	9.5
Absorber axial length	[cm]	24.00	27.13

Table: Parameters of the open cavity ring and the RFOFO ring.

Magnetic field profiles

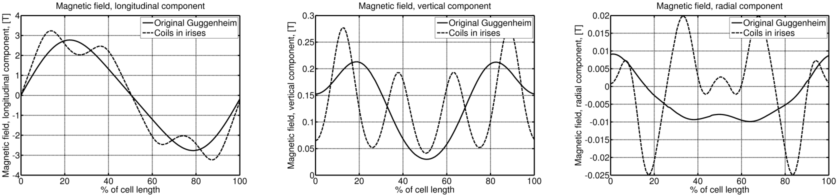


Figure: Longitudinal, vertical and radial components of the magnetic field. Solid line—original RFOFO ring (or Guggenheim helix), dashed line—open cavity lattice.

Closed orbits

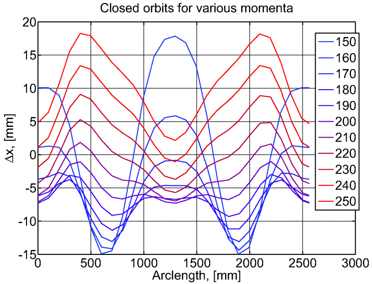


Figure: Closed orbit horizontal and vertical offsets along one cell of the cooling channel (2560 mm) for various momenta from 150 MeV/c to 250 MeV/c.

Tracking results

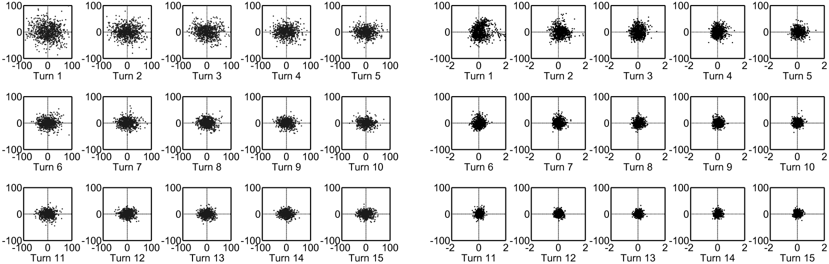


Figure: Phase portraits in the $(x - p_x)$ (left) and $(t - p_z)$ (right) planes, decay and stochastic processes on. The beam emittance is reduced until the equivilibrium emittance is reached.

Performance comparison: RFOFO vs. open cavity

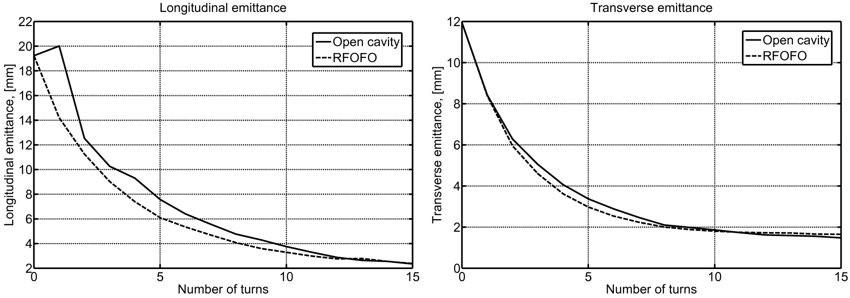


Figure: Performance of the open cavity lattice *vs.* the RFOFO lattice with decay and stochastic processes. Solid line—open cavity lattice, dashed line—RFOFO lattice.

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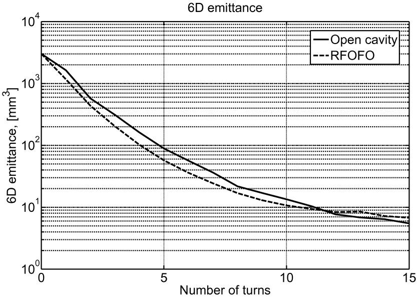

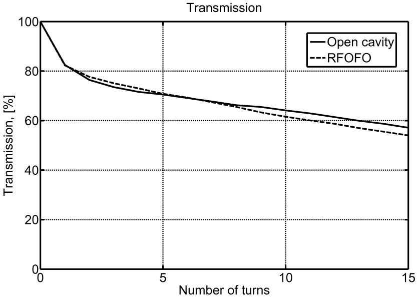

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Quantitative analysis of open cavity lattice vs. RFOFO

Structure	$\varepsilon_\perp$ [mm]	$\varepsilon_{\parallel}$ [mm]	ε_{6D} [mm^3]	Transmission [%]
Initial	12	19	3000	100
Open cavity (15 turns)	1.5	2.3	5.5	57
RFOFO (14 turns)	1.7	2.5	7.2	56
RFOFO (15 turns)	1.6	2.4	6.7	54

Table: Parameters of the open cavity ring compared to the RFOFO ring.

Summary

- MC and NF cooling needs are summarized.
- RFOFO and Guggenheim study results are presented.
- Open cavity lattice simulation results are summarized and compared to the RFOFO lattice.

P.S. I have some extra slides comparing tracking of the open cavity ring with and without decay/stochastic processes on.

Extra slides

Decay/stochastics on and off

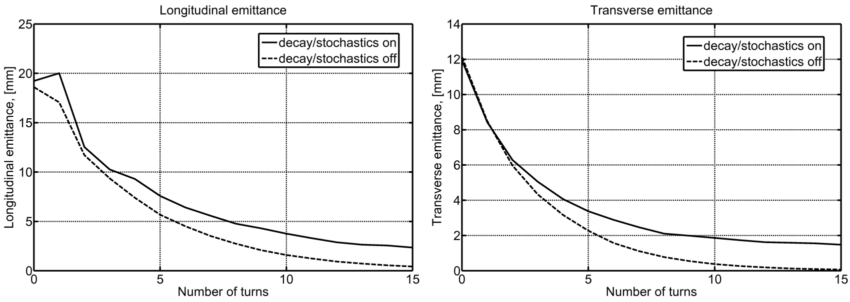

Figure: Performance of the open cavity lattice with decay and stochastic processes. Solid line—decay/stochastics on, dashed line—decay/stochastics off.

- There is no equilibrium emittance when stochastic processes are off, both transverse and longitudinal emittances shrink to zero

Decay/stochastics on and off

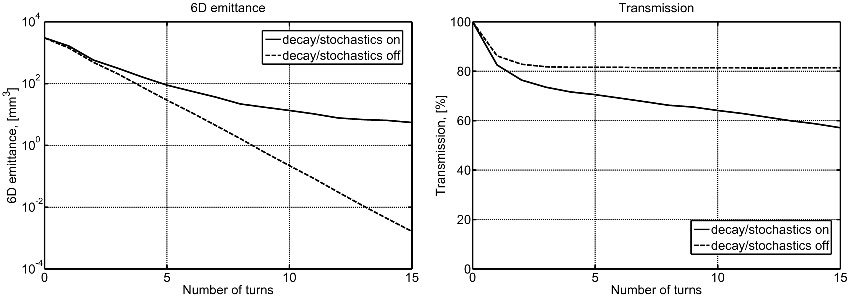

Figure: Performance of the open cavity lattice with decay and stochastic processes. Solid line—decay/stochastics on, dashed line—decay/stochastics off.

- With no stochastics the 6D emittance shrinks exponentially.
- With no decay the transmission stabilizes after 3 turns at 81%.